



# Proceedings of the fourteenth Congress of the European Society for Research in Mathematics Education

**Editors:** Marianna Bosch Casabò, Susana Carreira, Giorgio Bolondi, Michael Gaidoschik and Camilla Spagnolo  
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